HAC Covariance Matrix
(Heteroskedasticity and Autocorrelation Consistent)

Newey-West HAC 考慮了迴歸殘差未知形式的異質變異和序列相關。
White 則只有異質變異

$$\hat{\Omega}_{HAC} = \hat{\Gamma}(0) + \left(\sum_{j=1}^{T-1} \kappa(j,q)(\hat{\Gamma}(j) - \hat{\Gamma}'(j)) \right)$$

where $\hat{\Gamma}(0) = \frac{1}{T-k}\left(\sum_{t=j+1}^{T} Z_t' u_t u_t' Z_t \right)$ and $\hat{\Gamma}(j) = \frac{1}{T-k}\left(\sum_{t=j+1}^{T} Z_{t-j}' u_t u_{t-j}' Z_t \right)$,

u_t 是殘差向量，Z_t 是 a k×p 矩陣，滿足在時間 t 時 p 個動差條件：

$$m(\theta, y_t, X_t, Z_t) = Z_t' u(\theta, y_t, X_t)$$

kernel options: κ

Kernel 使用的目的在於對共變異數矩陣加權，以使共變異數矩陣為半正定（semi positive definite），確保非負特徵值（non-negative eigenvalue）。 EVIEWS 有兩種選擇：

(1) Bartlett kernel: $\kappa(j,q) = \begin{cases} 1-(j/q) & 0 \le j \le q \\ 0 & \text{otherwise} \end{cases}$

(2) Quadratic kernel: $\kappa(j/q) = \frac{25}{12(\pi x)^2}\left(\frac{\sin(6\pi x/5)}{6\pi x/5} - \cos(6\pi x/5) \right)$

註：QS kernel 比 Bartlett kernel 收斂速度要快，較平滑且也沒有被截斷（truncated）。

Bandwidth Selection: q

(1) The Newey-West fixed bandwidth:

$$q = \text{int}(4(\frac{T}{100})^{2/9})$$

頻寬（bandwidth）決定了估計共變異數矩陣時，*kernel* 函數中因落後期改變而調整共變異數矩陣的權重（weights）。EVIEWS 提供兩種自動選擇頻寬的函數，由資料「自相關」（autocorrelation）決定。

$$q = \begin{cases} \text{int}(1.1447(\hat{\alpha}(1)T)^{1/3}) & \text{for the Bartlett kernel} \\ 1.3221(\hat{\alpha}(2)T)^{1/5} & \text{for the QS kernel} \end{cases}$$

$\hat{\alpha}$ 的計算有兩種方法，如下：

（一）Andrews (1991)

$$\hat{\alpha}(1) = \frac{\left(\sum_{i=1}^{zn} \frac{4\hat{\rho}_i^2 \hat{\sigma}_i^4}{(1-\hat{\rho}_i)^6 (1+\hat{\rho}_i)^2}\right)}{\left(\sum_{i=1}^{zn} \frac{\hat{\sigma}_i^4}{(1-\hat{\rho}_i)^4}\right)} \qquad \hat{\alpha}(2) = \frac{\left(\sum_{i=1}^{zn} \frac{4\hat{\rho}_i^2 \hat{\sigma}_i^4}{(1-\hat{\rho}_i)^8}\right)}{\left(\sum_{i=1}^{zn} \frac{\hat{\sigma}_i^4}{(1-\hat{\rho}_i)^4}\right)}$$

（二）Newey-West 無母數方法：Based on a truncated weighted sum of the estimated cross-moments $\hat{\Gamma}(j)$.

$$\hat{\alpha}(p) = \left(\frac{l'F(p)l}{l'F(0)l}\right)$$

where l is unit vector and $F(p) = \Gamma(0) + \sum_{i=1}^{L} i^p (\hat{\Gamma}(i) + \hat{\Gamma}'(i))$.

Newey-West 無母數方法必須選擇一個落後期數，EVIEWS 提供兩種選擇：

$$L = \begin{cases} \text{int}(4(T/100)^{2/9}) & \text{for the Bartlett kernel} \\ T & \text{for the QS kernel} \end{cases}$$

Prewhitening

$m_t = Am_{t-1} + v_t$

$u'Z\hat{\Omega}^{-1}Z'u$